CS 421 --- Small Step Semantics Activity

Manager	Keeps team on track	
Recorder	Records decisions / QC	
Reporter	Reports to class	
Reflector	Assesses team performance	

Please write your name/netid legibly in dark ink. Hand in one copy per team. Do not staple or mangle the corners.

Purpose

Describing precisely what a programming language construct means mathematically allows you to reason about it, implement it, and understand its properties. This activity will help you explore transition semantics, a.k.a. small step semantics.

Your objectives:

- Identify the correct semantic rule to apply to a given program.

- Use the semantic rules to determine the value of a program.

- Demonstrate the Church Rosser Property and check if two reductions systems have it.

Part 1 --- Know the Rules

Here are the rules presented in the video.

$$\langle \texttt{skip}, \sigma \rangle \rightarrow \langle E, \sigma \rangle$$
$$\langle u := t, \sigma \rangle \rightarrow \langle E, \sigma[u := \sigma(t)] \rangle$$

$$\frac{\langle S_1, \sigma \rangle \rightarrow \langle S_2, \tau \rangle}{\langle S_1; S, \sigma \rangle \rightarrow \langle S_2; S, \tau \rangle}$$

$$E; S \equiv S$$

$$\langle \texttt{if } B \texttt{ then } S_1 \texttt{ else } S_2 \texttt{ fi}, \sigma \rangle \rightarrow \langle S_1, \sigma \rangle \quad \text{where } \sigma \models B$$
$$\langle \texttt{if } B \texttt{ then } S_1 \texttt{ else } S_2 \texttt{ fi}, \sigma \rangle \rightarrow \langle S_2, \sigma \rangle \quad \text{where } \sigma \models \neg B$$

$$\langle \texttt{while } B \texttt{ do } S_1 \texttt{ od}, \sigma \rangle \rightarrow \langle S_1; \texttt{while } B \texttt{ do } S_1 \texttt{ od}, \sigma \rangle \quad \text{where } \sigma \models B$$
$$\langle \texttt{while } B \texttt{ do } S_1 \texttt{ od}, \sigma \rangle \rightarrow \langle E, \sigma \rangle \quad \text{where } \sigma \models \neg B$$

Problem 1) What is the difference between E, S, and S_1 in the above rules? What does it mean $E; S \equiv S$?

Problem 2) In the first `while` rule, there is the state $\langle S_1; \texttt{while } B \texttt{ do } S_1 \texttt{ od}, \sigma \rangle$ where $\sigma \models B$
Why are there two copies of S_1?

Part 2 --- Reductions

Reduce the following programs according to the semantic rules given.

Problem 3)

⟨ if x>y then m:=x else skip fi; if x<y then m:=y else skip fi, {x:=10,y:=30}⟩

Problem 4)

⟨ n := 0; while x > 1 do x:=x/2; n:=n+1 od, {x:=8}⟩

Problem 5)

(Don't spend too much time on this one.)

⟨p:=1; n:=3; while n>1 do p:=p*x od, {x:=3}⟩

Part 3 --- Make your own rules!

Problem 6)

Write a rule to explain the `when B do S od` statement. It executes S only if B is true.

Problem 7)

Write a rule for `do S while B od`. It is like while, but executes S at least one time.

Problem 8) Let's add multitasking! Write a semantic rule[1] for `S1 || S2` which means that one of S_1 or S_2 can take a step.

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[1]Hm... maybe you will need more than one rule....

Part 4 --- Church Rosser

Problem 9) Consider the following definition of ∘:

$$x_1 \circ x_2 \circ \cdots \circ x_i \circ x_{i+1} \circ \cdots x_n \to x_1 \circ x_2 \circ \cdots \circ (x_i * x_{i+1}) \circ \cdots x_n$$

Does it have the Church-Rosser property? Try to prove it.

Problem 10) Consider the following definition of ∘:

$$x_1 \circ x_2 \circ \cdots \circ x_i \circ x_{i+1} \circ \cdots x_n \to x_1 \circ x_2 \circ \cdots \circ (x_i - x_{i+1}) \circ \cdots x_n$$

Does it have the Church-Rosser property? Try to prove it.

Small Step Semantics Activity --- Team's Assessment (SII)

Manager or Reflector: Consider the objectives of this activity and your team's experience with it, and then answer the following questions after consulting with your team.

1. What was a **strength** of this activity? List one aspect that helped it achieve its purpose.

2. What is one things we could do to **improve** this activity to make it more effective?

3. What **insights** did you have about the activity, either the content or at the meta level?

Small Step Semantics Activity--- Reflector's Report

Manager	Keeps team on track	
Recorder	Records decisions	
Reporter	Reports to Class	
Reflector	Assesses team performance	

1. What was a strength of your team's performance for this activity?

2. What could you do next time to increase your team's performance?

3. What insights did you have about the activity or your team's interaction today?