CS 421 --- Ambiguous Grammars / First and Follow Sets / Solutions
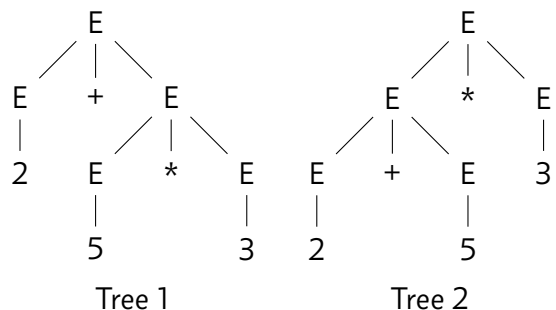## Mattox Beckman

Part 1 --- Ambigous Grammars

These grammars are ambiguous. Prove that they are ambiguous by giving an input with two parse trees. Give an equivalent grammar that is unambiguous.

Example 1)

$$
\begin{aligned}
E \rightarrow \ & E \ + \ E \\
| \ & E \ * \ E \\
| \ & - E \\
| \ & i
\end{aligned}
$$

This grammar is ambiguous. Consider the input `2 + 5 * 3`. (Let `i` be an integer.) There are two possible trees for this:

```
            E                          E
         ╱  |  ╲                    ╱  |  ╲
      E    +    E                E    *    E
      |      ╱ | ╲               ╱ | ╲      |
      2    E   *   E           E   +   E    3
           |       |           |       |
           5       3           2       5
          Tree 1                      Tree 2
```
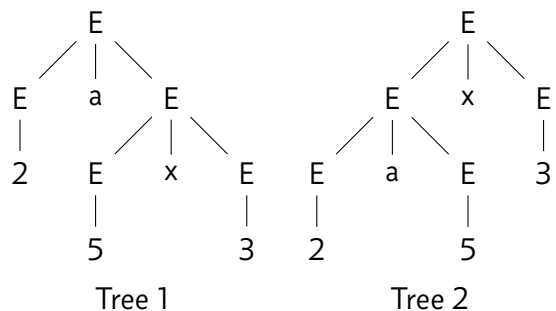
Rewrite the grammar so that `*` and `+` have their usual precedences, and associate to the left. Unary minus binds most tightly.

$$
\begin{aligned}
E \rightarrow \ & E \ + \ T \\
| \ & T \\
T \rightarrow \ & T \ + \ F \\
| \ & F \\
F \rightarrow \ & - F \\
| \ & i
\end{aligned}
$$

Problem 1)

$$
\begin{aligned}
E \rightarrow \ & E \ a \ E \\
| \ & E \ b \ E \\
| \ & E \ x \ E \\
| \ & E \ y \ E \\
| \ & i
\end{aligned}
$$

The grammar is ambiguous. Here are two trees that demonstrate an ambiguity, for input `2 a 5 x 3`.

```
            E                          E
         ╱  |  ╲                    ╱  |  ╲
      E    a    E                E    x    E
      |      ╱ | ╲               ╱ | ╲      |
      2    E   x   E           E   a   E    3
           |       |           |       |
           5       3           2       5
          Tree 1                      Tree 2
```

Rewrite the grammar so that a has highest precedence, associating to the left. Next is b, associating to the left. Then we have y associating to the right. Let x have the lowest precedence, and associate to the right.

$$E \rightarrow F\,x\,E$$
$$\mid F$$
$$F \rightarrow G\,y\,F$$
$$\mid G$$
$$G \rightarrow G\,b\,H$$
$$\mid H$$
$$H \rightarrow H\,a\,i$$
$$\mid i$$

Part 2 --- First and Follow Sets

Calculate the first and follow sets for these grammars.

Example 2)
$$S \rightarrow a\,E\,b$$
$$\mid x$$
$$E \rightarrow x\,y$$
$$\mid \epsilon$$

Symbol	First	Follow
S	a, x	$\$$
E	x, ϵ	b

Example 3)
$$S \rightarrow a\,E\,F\,b$$
$$\mid x\,E\,F$$
$$E \rightarrow x\,y$$
$$\mid \epsilon$$
$$F \rightarrow E\,z\,q$$
$$\mid w\,S$$

Symbol	First	Follow
S	a, x	$b, \$$
E	x, ϵ	x, w, z
F	x, w, z	$b, \$$

Problem 2)
$$S \rightarrow a\,F\,E\,b$$
$$\mid x\,F\,E$$
$$E \rightarrow x\,y$$
$$\mid \epsilon$$
$$F \rightarrow E\,z\,q$$
$$\mid w\,S$$

Symbol	First	Follow
S	a, x	$b, x, \$$
E	x, ϵ	$b, x, z, \$$
F	x, w, z	$b, x, \$$